Papers written by Australian Maths Software

SEMESTER ONE

YEAR 12

MATHEMATICS METHODS

REVISION 1

Unit 3

2016

Section Two

(Calculator–assumed)

Name: _____

Teacher: _____

TIME ALLOWED FOR THIS SECTION

Reading time before commencing work: 10 minutes
Working time for section: 100 minutes

MATERIAL REQUIRED / RECOMMENDED FOR THIS SECTION

To be provided by the candidate
Standard items*:* pens, pencils, pencil sharpener, highlighter, eraser, ruler.

Special items: drawing instruments, templates, notes on up to two unfolded sheet of A4 paper, and up to three calculators approved for use in the WACE examinations.

IMPORTANT NOTE TO CANDIDATES

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non–personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

To be provided by the supervisor

Question/answer booklet for Section Two.
Formula sheet retained from Section One.

Structure of this examination

	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One Calculator—free	7	7	50	50	35
Section Two Calculator—assumed	**13**	**13**	**100**	**100**	**65**
			Total marks	150	100

Instructions to candidates

1. The rules for the conduct of this examination are detailed in the Information Handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in the Question/Answer booklet.

3. You must be careful to confine your responses to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Spare pages are provided at the end of this booklet. If you need to use them, indicate in the original answer space where the answer is continued i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer booklet.

8. (4 marks)

The volume of a cubical ice block is 1cm^3.

(a) Write the expression for the volume of a cubical ice block of side x. (1)

(a) Use a calculus method to determine the decrease in volume when the side has melted to 9 mm. (3)

9. (10 marks)

(a) Which of the following tables could represent a probability density function? Give your reasons.

(i)

x	1	2	3	4
$P(X=x)$	0.2	0.2	0.3	0.2

(ii)

x	5	6	7	8
$P(X=x)$	0.2	0.4	0.5	-0.1

(iii)

x	0	1	2
$P(X=x)$	0.3	0.4	0.3

(iv)

x	10	20	30	40
$P(X=x)$	0.3	0.2	1.1	03

(1+1+1+1)

(b) A die is rolled. The outcomes form the set Y.

(i) Complete the following probability table for the discrete random variable Y. (1)

y	1	2	3	4	5	6
$P(Y = y)$						

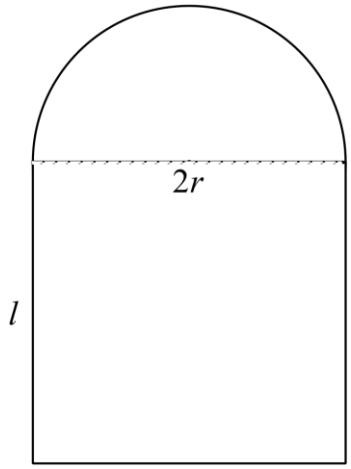
(ii) Find $P(Y \leq 4)$. (1)

(ii) Determine the expected value and standard deviation of Y. (4)

10. (8 marks)

Joe is designing a window with a semi-circular feature at the top as in the diagram below. Joe's client insists that the perimeter of the window is exactly 4 m.

The client also wants to maximise the window area so he obtains as much light as possible.

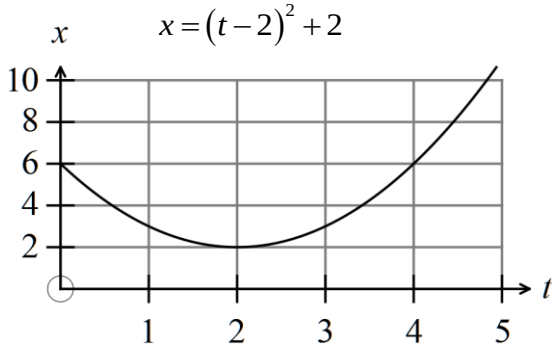


(a) Find an expression for the perimeter of the window. (1)

(b) Find an expression for the area of the window in terms of r. (2)

(c) Use a calculus method to find the maximum area of the window.
Show all working. (5)

11. (6 marks)



The graph illustrates the motion of a particle for $t \geq 0$ where x represents displacement in metres and t represents time in seconds.

(a) State when the particle changes direction. (1)

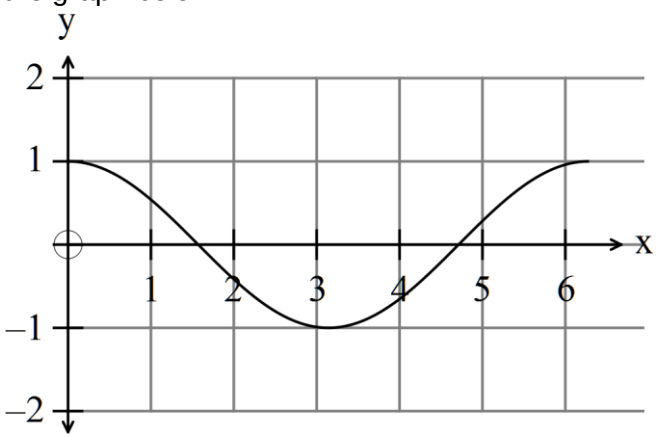
(b) State when the acceleration is positive and when the acceleration is negative. Confirm your statement by finding an expression for the acceleration. (2)

(c) Find the velocity at $t = 3$. (1)

(d) Find the distance travelled for $1 \leq t \leq 4$. (2)

12. (7 marks)

(a) Consider the graph below.



(i) Use the graph to estimate the gradient at different points to help you plot the gradient function on the same set of axes. (3)

(ii) Mark clearly the point on the graph where the gradient is maximum. (1)

(b) Given the function $f(x) = cos(x)$.

Determine the derivative of the function f. (1)

(c) Use your calculator to estimate $\lim_{h \to 0} \left(\frac{e^h - 1}{h} \right)$. (2)

13. (5 marks)

(a) Evaluate $\int_0^{\pi/8}\left(\frac{sin(2x)}{1+2x}\right)dx$ correct to four decimal places. (2)

(b) Given $f(x)=e^x\times sin(x)$

(i) find $f'(x)$. (1)

(ii) hence find $y=\int e^x\left(sin(x)+cos(x)\right)dx$ given $y=1$ when $x=0$. (2)

14. (9 marks)

The velocity of a body is $v = -3t + 6$ m/s and the initial displacement is $2m$.
Assume $t \geq 0$.

(a) Find the expression for the displacement. (2)

(b) Find the expression for the acceleration. (1)

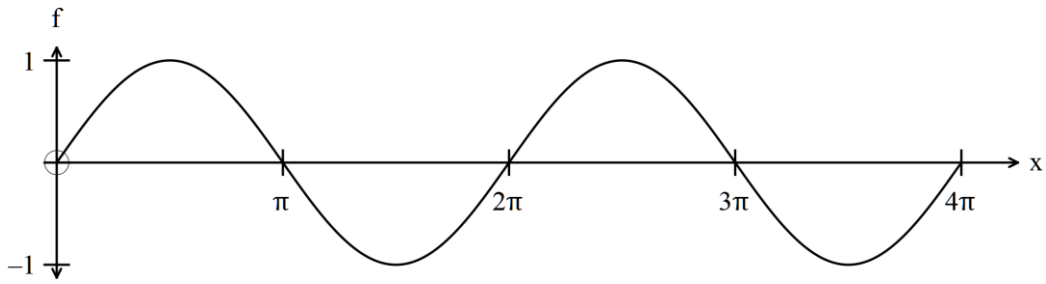
(c) Find the second time when the displacement is equal to the initial displacement. (2)

(d) Find the displacement when the body changes direction. (2)

(e) How far does the body travel in the first second after the body changes direction? (2)

15. (13 marks)

(a) Consider the graph below



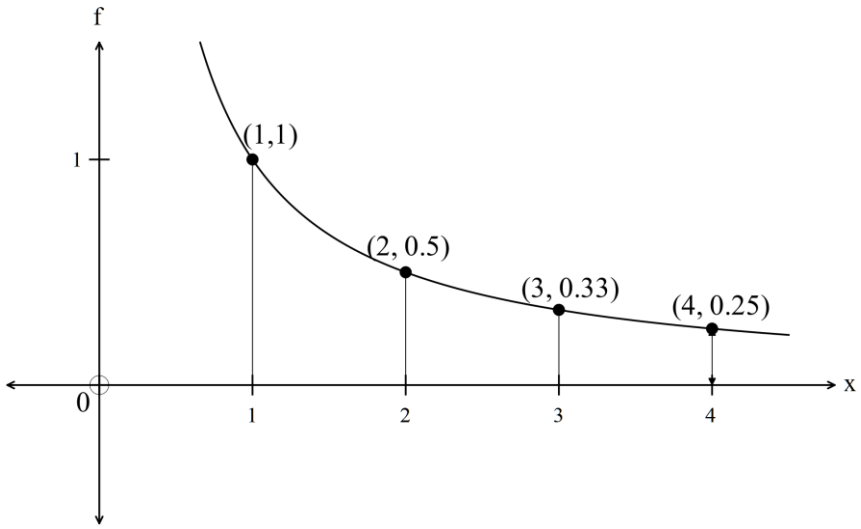
(i) Evaluate $\int_0^{\pi} sin(x)\,dx$. (1)

(ii) Hence estimate $\int_0^{\pi/2} sin(x)\,dx$ and explain why an estimate can be made. (1)

(iii) Find the area enclosed by the function and the x axis on $[0,4\pi]$. (1)

(iv) Evaluate $\int_0^{4\pi} sin(x)\,dx$. (1)

(b) Consider the function $f(x)=\frac{1}{x}$ graphed below:



(i) Estimate the area between the function $y=f(x)$, the x axis and $x=1$ and $x=4$ using rectangles from below and from above. (4)

(ii) Explain why $\int_1^4 \frac{1}{x}dx$ cannot be readily calculated using conventional methods. (3)

(iii) Use your calculator to determine $\int_1^4 \frac{1}{x}dx$ correct to 3 decimal places. (2)

16. (8 marks)

In 2000, the population of numbats in a small park was close to 400. A section of the park was sealed off to protect the species from feral cats etc. In 2008, the population had increased to 550.

(a) What is the annual rate of increase of the numbats over that time? (3)

(b) Estimate the number of numbats in 2016. (2)

It was decided to leave the section of the park sealed off to protect the numbats.

(c) If in 2020 the population was 780, is it possible that the predators had found a way into the parklands? Give your reasons. (3)

17. (10 marks)

(a) The local high school uses a house system The houses are named after various recent prime ministers, Hawke (Labor), Howard (Liberal), Gillard (Labor), and Turnbull (Liberal).
A class of 20 students was split between the houses as follows.

House	Hawke	Howard	Gillard	Turnbull
Number of students	3	5	7	5

One of the students is selected at random from the class.

(i) Draw up a probability density table for the probability of being in a particular house. (2)

House	Hawke	Howard	Gillard	Turnbull
P(House)				

(ii) If one of the students is selected at random, what is the probability that the student was in a house named after a Labor prime minister? (2)

(b) The school has 800 students and exactly 200 of them are in each house.

(i) Draw up a probability density table for the probability of being in a particular house using the whole student population. (2)

House	Hawke	Howard	Gillard	Turnbull
P(House)				

(ii) Explain the differences in the tables in (a)(i) and (b)(i). (2)

A new student is assigned to a house at random. What is the probability that

(iii) the student is assigned to Howard or Hawke house? (1)

(iv) the student is not assigned to Turnbull house? (1)

18. (9 marks)

The probability of Anne receiving mail on any business day is estimated to be about 70%. Assume there are 5 business days in a week.

(a) What is the probability that Anne receives no mail on any given business day in a particular week? (1)

(b) What is the probability that Anne receives mail on at least 3 days of any given 5 business days? (2)

(c) Given that Anne received mail on Monday, what is the probability that she receives mail on exactly 3 business days that week? (2)

(d) What is the average number of days per week that Anne receives mail? (2)

(e) Calculate the standard deviation of the distribution. (2)

19. (6 marks)

(a) Two coins are tossed. Wrote down the probabilities in the following table. . (2)

x	HH	HT or TH	TT
$P(X = x)$			

(b) If two coins are tossed 50 times, what is the probability that

(i) two heads appear 10 times? (1)

(ii) two heads appear between 10 and 40 times inclusive? (1)

(iii) there is at least one tail at least 30 times? (2)

20. (5 marks)

A company office has seven laptops - four Apple MacBooks, each of which is in use, on average, 70 percent of the time, and three ASUS, each of which is in use, independently of the Macbooks, 80 percent of the time.

If the manager walks into the office at any time, what is the probability that three MacBooks and the three ASUS will be in use? (5)

END OF SECTION TWO